

*Corresponding author: Khadijah,
Pendidikan Matematika, Fakultas
Matematika dan Ilmu Pengetahuan
Alam, Universitas Negeri Makassar,
Makassar, Indonesia.

E-mail: khadijah@unm.ac.id

RESEARCH ARTICLE

Analysis of Higher Order Thinking Skills in Exponents And Logarithms

Khadijah*, Irfan Pratama Amrin, & Muh. Zainal Pratama

Pendidikan Matematika, Fakultas Matematika dan Ilmu Pengetahuan Alam,
Universitas Negeri Makassar, Makassar, Indonesia.

Abstract: This study is intended to investigate the subject of exponent, roots and logarithms and how teacher can help students understand this subject better. The study method displays several facts about Exponent, Roots and Logarithm, and the correlation of them in daily life. Then, in every part, it will derive and prove the Properties of Exponent, Roots and Logarithm. Analyzing the impact of lack of concept understanding on students' ability to solve mathematics problems. Identifying common errors in learning exponent, root, and logarithm forms material. Propose strategies and ways to avoid them. Presenting effective learning media for exponent, roots and logarithm. And outlining the benefits of using UN type questions in improving understanding and Higher Order Thinking Skills (HOTS). The result of this study, we found that the Exponent, Roots and Logarithm are important foundation for various applications in daily life and in higher mathematics subjects. To assist learners in learning this material, the paper also presents a range of suggested learning media. These include the use of manipulatives, interactive whiteboards, learning videos, and worksheets. These media will help learners to visualize the concepts and apply them in different contexts. Finally, the paper suggests using UN-type questions (HOTS) to test learners' understanding. These problems are designed to encourage learners to think at a higher level in mathematical problem solving, so that they can better master the concepts of exponents, roots and logarithms.

Keywords: Exponent, Logarithm, Higher Order Thinking Skills (HOTS), Roots, Learning Package.

1. Introduction

Mathematics is a universal language that has an important role in the development of science and technology. One of the important aspects of mathematics is the material on exponent, roots, and logarithms. This material is fundamental to many fields of science, including physics, computer science, statistics, and engineering. The exponential and logarithmic functions are essential building blocks of classical calculus (Larsson & Ruf, 2019). In addition, a strong understanding of these concepts is also very relevant in everyday life, especially in problem solving and decision making.

However, the subject of exponent, roots, and logarithms is often considered difficult by most students. Some of the main underlying reasons are that concepts in exponents, roots and logarithms are often considered difficult to understand due to their abstract nature, as is the case with the concepts of exponents and logarithms, which can be complicated for students who do not yet have a solid understanding of the basics of mathematics. In addition, available



learning resources, especially textbooks, sometimes lack adequate explanations or sufficient examples to help students understand the material well. Related to this, students also often face difficulties in relating these mathematical concepts to real-world situations, which can lead to confusion and not motivated in the learning process. In addition, students often get caught up in misunderstanding basic concepts, which can hinder a deeper understanding of this material.

Therefore, we need to further investigate the subject of exponent, roots and logarithms and how we can help students understand this subject better. New method in logarithms performs much better than the most widely known algorithm (Hong & Lee, 2019). This paper will discuss important aspects of this material, recognize common problems that students face, and look for ways to teach and understand this material better. With the hope that a better understanding will make students feel more confident and able to use these mathematical concepts in daily life.

The new index calculus algorithm that exploits summation polynomials for solving the discrete logarithm problem on elliptic curves defined over finite fields is founded, but it needs to be deeply analyze (Amadori et al., 2018). From it, we know that we need a skill of deeply analyze to found the solution of a problem.

At the end of phase E in the curriculum generally aimed at grade X SMA/MA/Package C program, learners have achieved a number of important milestones in mathematics learning. They have succeeded in generalizing the properties of the operations of exponent numbers or exponents, and are able to apply them in solving various problems related to exponential equations that have the same basis. In addition, learners can also understand the concept of exponential functions well.

Not only that, learners have also developed the ability to generalize the properties of exponent and logarithm operations. They are able to quickly digest and understand these concepts, thus achieving higher order thinking skills (HOTS) in mathematical problem solving. This ability is not only limited to application in learning contexts, but also helps learners to develop leadership skills that are valuable in everyday life.

These achievements reflect learners' commitment to mastering complex and relevant mathematical material. This learning outcome can be a valuable contribution to an in-depth paper on mathematics curriculum development in higher education.

Problem Formulation

- (1). What is the importance of understanding the concepts of exponents, roots, and logarithms in mathematics?
- (2). What impact does the lack of understanding of these concepts have on students' ability to solve math problems?
- (3). What are some common mistakes that often occur when students learn about exponents, roots, and logarithms?
- (4). How to avoid common mistakes in learning this material?
- (5). What is an effective learning media that can help students understand and apply these concepts better?
- (6). How can the use of un-type problems (HOTS) help improve students' understanding of the concepts of exponent, root and logarithm forms and higher order thinking skills in mathematical problem solving?

Purpose of The Paper

- (1). To describe the importance of understanding the concept of exponent, root, and logarithm forms in mathematics.
- (2). Analyzing the Impact of Lack of Concept Understanding on Students' Ability to Solve Mathematics Problems

- (3). Identifying Common Errors in Learning Exponent, Root, and Logarithm Forms Material
- (4). Propose Strategies and Ways to Avoid Common Errors in Learning
- (5). Presenting Effective Learning Media
- (6). Outlining the Benefits of Using UN (HOTS) Type Questions in Improving Understanding and High Level of Thinking Skill.

2. Research Method and Materials

The study is designed, and the results are reported, in accordance with the preferred reporting items for systematic reviews. The outline of paper such as, the first, we recall some facts about Exponent, Roots and Logarithm, and the correlation of them in daily life. Then, in every part, we will derive a Properties of Exponent, Roots and Logarithm (Chung, 2023). We will then prove the properties. Analyzing the impact of lack of concept understanding on students' ability to solve mathematics problems. Identifying common errors in learning exponent, root, and logarithm forms material. Propose strategies and ways to avoid common errors in learning. Presenting effective learning media for exponent, roots and logarithm. And outlining the benefits of using UN (HOTS) type questions in improving understanding and High Level of Thinking Skill.

3. Results and Discussion

In 2020, the world is faced with a Covid-19 virus outbreak that is spreading in almost all countries in the world. In Indonesia in 2020, Covid-19 transmission cases are still quite high and have not shown a significant decrease, even tending to increase. At the beginning of its transmission, the graph of the development of Covid-19 transmission was described as an exponential form. The exponential shape describes the situation of a rapid increase in a quantity in a certain period of time. Why is this the case? What is the exponential shape?

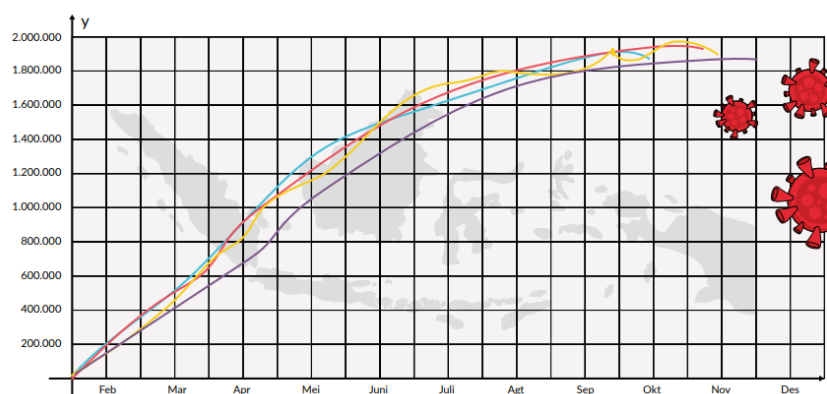


Figure 1. Exponential Graph of Covid-19 Spread

In addition, to observe the growth of bacteria or viruses, researchers usually observe how many bacteria will grow each hour. Researchers are able to predict how many bacteria will grow at certain hours with mathematical calculations or otherwise determine the time it takes for a certain number of bacteria to grow.



Figure 2. Exponential Graph of Covid-19 Spread

Similarly, to predict the population of an area a few years later, mathematical calculations can be used to determine it. By only making observations, it is certainly not easy. Certain calculations are needed to determine it.

How do learners think these problems can be solved mathematically? Exponents and logarithms are mathematical concepts that have an important role to play in solving problems like the ones mentioned above. How can you use these two concepts to solve problems like the ones above? And in what other contexts can these two concepts be used?

Learning exponentials can be challenging for many, and there are some common mistakes that often occur during the learning process. Ignoring the Relationship with Logarithms is one of the most common mistakes, ignoring the concept of logarithms can make you struggle when solving problems involving exponentials. Exponential and logarithm are closely related concepts. Therefore, we present a concept map so that we don't ignore the relationship between the two.



Figure 3. Concept Map

3.1. Exponents

3.1.1. Definition of Exponent

If a is a real number and n is a positive integer, then a^n denotes the product of the number a by n factors and is written as

$a^n = a \times a \times \dots \times a$ where there is repeated multiplication of a by n factors

There are some common mistakes that often occur during the learning process, one of which is misunderstanding or applying the rules of exponential operations, such as the exponent multiplication rule with the same base $a^m \cdot a^n = a^{m+n}$ the exponent division rule with the same base $\frac{a^m}{a^n} = a^{m-n}$, and so on. With that in mind, here are some important definitions that you need to know

If a is a number with $a \neq 0$ and n is a positive integer, then $a^{-n} = \left(\frac{1}{a}\right)^n$

If a is a real number with $a \neq 0$ and n is a positive integer, then $a^{\frac{1}{n}} = p$ is a positive real number, so $p^n = a$

If a is a real number with $a \neq 0$ and m, n are positive integers, then $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m$

Properties of exponents

Here are the properties of exponents that you need to know

- (1). $a^m \cdot a^n = a^{m+n}$, with $a \neq 0$, m, n integers
- (2). $\frac{a^m}{a^n} = a^{m-n}$, with $a \neq 0$, m, n integers
- (3). $(a^m)^n = a^{m \times n}$ with $a \neq 0$, m, n integers
- (4). $(ab)^m = a^m \times b^m$ with $a, b \neq 0$, and m an integer
- (5). $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ with $b \neq 0$, and m an integer
- (6). $\left(a^{\frac{m}{n}}\right)\left(a^{\frac{p}{n}}\right) = \left(a^{\frac{m+p}{n}}\right)$ with $a > 0$, $\frac{m}{n}$ and $\frac{p}{n}$ are rational numbers with $n, q \neq 0$
- (7). $\left(a^{\frac{m}{n}}\right)\left(a^{\frac{p}{q}}\right) = \left(a^{\frac{m+p}{n+q}}\right)$ with $a > 0$, $\frac{m}{n}$ and $\frac{p}{q}$ rational numbers with $n, q \neq 0$
- (8). $a^0 = 1$, with $a \neq 0$

Learning the properties of exponents can be challenging for students, this happens because some people sometimes ignore some of the properties of exponents. One of the exponential properties such as the law of exponents $a^{mn} = (a^m)^n$ or $a^0 = 1$ is usually forgotten by some people. In fact, these properties are very important to carry out correct algebraic manipulations. Therefore, students need to prove some exponential properties, so it is hoped that students can understand the concept of the properties of exponents well. Here are some proofs of the properties of exponents.

Proof of property 5

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \text{ with } b \neq 0, \text{ and } m \text{ integers}$$

$$\left(\frac{a}{b}\right)^m = \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \dots \times \frac{a}{b} \text{ where, there is a multiplication of } \frac{a}{b} \text{ by } m \text{ factors}$$

$$\left(\frac{a}{b}\right)^m = \frac{a \times a \times a \times \dots \times a}{b \times b \times b \times \dots \times b} \text{ where there is multiplication of } a \text{ and } b \text{ by } m \text{ factors} \rightarrow \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Proof of property 6

$$\left(a^{\frac{m}{n}}\right)\left(a^{\frac{p}{n}}\right) = \left(a^{\frac{m+p}{n}}\right) \text{ with } a > 0, \frac{m}{n} \text{ dan } \frac{p}{n} \text{ rational numbers with } n, q \neq 0$$

Proof

Using Property 1, it holds

$$\left(a^{\frac{m}{n}}\right)\left(a^{\frac{p}{n}}\right) = \left(a^{\frac{m+p}{n}}\right)$$

$$\left(a^{\frac{m}{n}}\right)\left(a^{\frac{p}{n}}\right) = \left(a^{\frac{m+p}{n}}\right)$$

Proof of property 7

$$\left(a^{\frac{m}{n}}\right)\left(a^{\frac{p}{q}}\right) = \left(a^{\frac{m+p}{n+q}}\right) \text{ dengan } a > 0, \frac{m}{n} \text{ dan } \frac{p}{q} \text{ rational numbers with } n, q \neq 0.$$

Proof:

By using Property 1 it holds

$$\left(a^{\frac{m}{n}}\right)\left(a^{\frac{p}{q}}\right) = \left(a^{\frac{m+p}{n+q}}\right)$$

Proof of property 8



$$a^0 = a^{n-n}$$

$$a^{n-n} = \frac{a^n}{a^n} = \frac{a \times a \times a \times \dots \times a}{a \times a \times a \times \dots \times a} = 1$$

3.1.2. Exponential Function

There are several common mistakes when students learn exponential function material, as for one of the mistakes that often arise is that students are unable to master the exponential graph, this happens because students have a lack of understanding of exponential growth. Lack of understanding of how exponential growth works, and how it occurs in various contexts, such as population growth or financial investment. This also leads to learners being unable to draw graphs of exponential functions, including understanding how parameters such as base and exponent affect the shape of the graph.

To overcome this, it is necessary for teachers to introduce the concept of exponential functions in a real context. Here we present a case of Introducing Exponential Growth through Real Situations

There is an event, where there is a person who carries a virus and infects 3 other people. In the next phase, each person infects 3 more people. Next, the questions that come to our mind are, "How many people will be infected in each subsequent phase?"

Consider the above case above.

In the first phase 3 people get infected from the first person and then transmit each to 3 other people. Then the 3 people transmit again to each of the next 3 people, and so on.

Table 1. Relation Phase and Transmit Virus

From Table 1, if learners notice, to determine the number of people who contract the virus,

Phase	1	2	3	4	5	6	7
Many people are infected	$3 = 3^1$	$9 = 3^2$	$27 = 3^3$	$81 = 3^4$	$243 = 3^5$	$729 = 3^6$	$2187 = 3^7$

the pattern that emerges is 3^x , where x is the phase of the virus spread. If $f(x)$ is the number of people who contract the virus, while x is the phase of the virus spread, then the number of people who contract the virus can be expressed by:

$$f(x) = 3^x$$

It can be concluded that $f(x) = 3^x$ is an example of an exponent function.

Definition of Exponent Function

An exponent function is expressed by

$$f(x) = n \times a^n$$

Where a is a cardinal number $a > 0, a \neq 1, n$ is a non-zero real number and x is any real number.

3.1.3. Exponential Growth

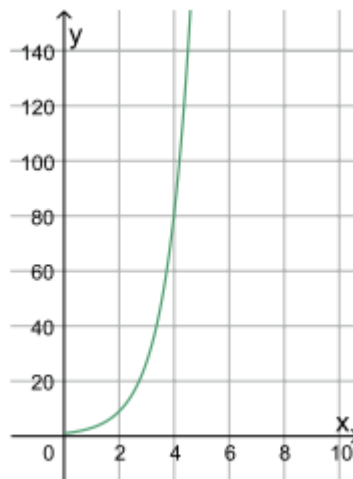


Figure 1. Function graph $f(x) = 3^x$

The curve above is one that shows exponential growth, where the growth rate is directly proportional to the value of the quantity. Another example is the growth of bacteria where in later phases the bacteria will certainly increase in number.

The exponential growth function is written as:

$$f(x) = a^x \text{ with } a > 1$$

3.1.4. Exponent Decay

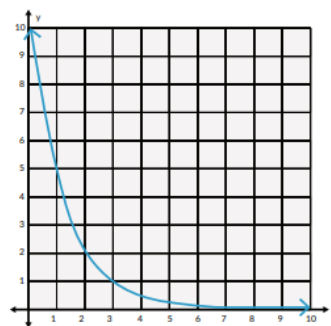


Figure 2. Exponential decay function graph

An exponent function not only describes significant growth over time. An exponent function also describes a consistent decline over a period of time. This is called exponential decay. Consider the graph of the exponent decay function below

An exponent decay function can be written as

$$f(x) = n \times a^x, \text{ with } 0 < a < 1, n \text{ is a nonzero real number, } x \text{ is any real number.}$$

3.2. Root Form

The most common condition for students when learning exponentials is that most students find it difficult and confusing to distinguish exponential forms from exponent forms. Exponential involves the power of a constant number to a base, while power involves repeated multiplication operations on the same number. Make sure you understand this fundamental difference. Therefore, it is important to emphasize the concept of the definition of the root form which is directly related to the exponent form

3.3. Logarithms

The condition that often occurs during the learning process of logarithm material is that students sometimes ignore the relationship between exponential and logarithm. Exponential and logarithm are concepts that are closely related. We can see the relationship between the general form of exponents and logarithms based on the definition of the logarithm itself.

Definition of Logarithm

Suppose a is a positive number with $0 < a < 1$ atau $a > 1, b > 0, \log_a b = c$ if and only if $b = a^c$

Where,

a is the base number or base logarithm

b is the numerus

c is the logarithm result

Definition of Common Logarithm

Logarithms that have base 10 are called generalized logarithms and are written as follows

$$\log_{10} a = \log a$$

Properties of logarithms

Suppose $a > 0$ and $a \neq 1, b > 0, m > 0, m \neq 1$, where a, b, c, m, n are Real numbers, then it applies

1. $\log_a a = 1$
2. $\log_a 1 = 0$
3. $\log_a a = n$
4. $\log_a (b \times c) = \log_a b + \log_a c$
5. $\log_a \left(\frac{b}{c}\right) = \log_a b - \log_a c$
6. $\log_a b^n = n \log_a b$
7. $\log_a b = \frac{\log_m b}{\log_m a} = \frac{1}{\log_b a}$
8. $\log_a b \times \log_b c = \log_a c$

Learning the properties of logarithms can be challenging for students, this happens because some people sometimes ignore some properties of logarithms. One of the properties of logarithms such as the logarithm law $\log(ab) = \log(a) + \log(b)$ or $\log(1) = 0$ is usually forgotten by some people. In fact, these properties are very important to carry out correct algebraic manipulations. Therefore, students need to prove some properties of logarithms, so it is hoped that students can understand the concept of the properties of logarithms well. Here is one proofs of the properties of logarithms.

Prove the logarithm property $\log_a (b \times c) = \log_a b + \log_a c$

Alternative Proof:

Suppose $\log_a b = m$ and $\log_a c = n$

We can write it in its exponent form as follows:

$$b = a^m \text{ dan } c = a^n$$

Recall the exponent property $a^m \cdot a^n = a^{m+n}$



$$a^m \cdot a^n = a^{m+n}$$

$$b \cdot c = a^{m+n}$$

$$\log_a(bc) = \log_a(a^{m+n})$$

$$\log_a(bc) = m + n$$

$$\log_a(bc) = \log_a b + \log_a c$$

3.4. Suggested Learning Media

In learning the concept of exponentials and logarithms, we want to make learning more interest and easy to understand. We have designed some learning media that will help us understand this concept better. Although with these challenges, computational thinking education could bring new learning opportunities and enhance children's computational thinking skills, as well as other non-cognitive skills such as critical thinking, body-material interaction, and hand-eye coordination (Su & Yang, 2023). The other ways, we can use a game-based learning environment. In a game-based learning environment that promotes the development of adaptive expertise with arithmetic, students can engage in practising their arithmetic skills at multiple levels of difficulty within the same task (Bui et al., 2023).

First, we will use manipulatives, such as candies or other small objects, to illustrate exponential growth. This will allow us to see how something can grow very fast and be easier to understand. Next, we will use an interactive whiteboard. This kind of approach of the teaching is desirable. The inclusion of interactivity and multimedia elements in the teaching serves as a convenient means of awakening interest in students and enhances students' self-logical thinking (Renata & Jana, 2012). The digital whiteboard will allow us to draw exponential graphs and formulas directly. This will help us see the relationship between the numbers and how exponentials work.

Then, we will use a learning video. In research of (Aksel & Gürman-Kahraman, 2014), they found that the students were in favour of projects' more contribution to the learning video. In these short videos, we will see how the concept of exponential growth is applied in real-world situations. This will help us connect theory with practice. In addition, we will also have worksheets and assessment sheets. This will help us organize our understanding of the exponential concept and see how far we have understood it.

Lastly, we will encourage collaboration between students through projects and presentations. This will help us gain a deeper understanding of the concept and how we can apply it in our daily lives. By using these various learning media, we will be able to better understand the concepts of exponentials and logarithms and apply them in real-life situations. Let's explore the exciting world of math together!

3.5. National Exam Practice Questions

Lack of practice and understanding of concepts is one of the events that very often happens to students. For this reason, in order for students to understand the concept of exponential well, they need repeated practice problems. Lack of practice and deep understanding can hinder the development of concept understanding. Therefore, we present some national exam-type problems, so that students are able to achieve higher order thinking skills (HOTS) in mathematical problem solving.

- (1). The value of $\frac{\frac{3}{2}a^2+4a^{-1}-5a^0}{b^3+3b^{-1}-2b^0}$ for $a = 4$ and $b = \frac{1}{8}$ is
- 88
 - $\frac{89}{4}$
 - $\frac{89}{88}$



d. $\frac{4}{89}$
e. $\frac{16}{89}$

Answer:

$$a = 2^2 \text{ and } b = 2^{-3}$$

$$= \frac{(2^2)^{\frac{3}{2}} + 4 \cdot (2^2)^{-1} - 5 \cdot 1}{(2^{-3})^{\frac{2}{3}} + 3 \cdot (2^{-3})^{-1} - 2 \cdot 1}$$

$$= \frac{2^3 + 2^2(2^{-2}) - 5}{2^{-2} + 3 \cdot 2^3 - 2}$$

$$= \frac{8 + 1 - 5}{\frac{1}{4} + 24 - 2}$$

$$= \frac{4}{\frac{1}{4} + 22}$$

$$= \frac{4}{\frac{1}{4} + \frac{88}{4}}$$

$$= \frac{4}{\frac{89}{4}}$$

$$= 4 \cdot \frac{4}{89}$$

$$= \frac{16}{89}$$

(2). A simple form of $\frac{(\sqrt{3} + \sqrt{7})(\sqrt{3} - \sqrt{7})}{2\sqrt{5} - 4\sqrt{2}}$ is

- a. $\frac{2}{3}(\sqrt{5} + 2\sqrt{2})$
b. $-\frac{2}{3}(2\sqrt{5} + 4\sqrt{2})$
c. $-\frac{4}{9}(2\sqrt{5} + 4\sqrt{2})$
d. $\frac{2}{3}(2\sqrt{2} - \sqrt{5})$
e. $-\frac{4}{9}(2\sqrt{5} + 4\sqrt{2})$ Answer:

$$\frac{(\sqrt{3} + \sqrt{7})(\sqrt{3} - \sqrt{7})}{2\sqrt{5} - 4\sqrt{2}} \times \frac{2\sqrt{5} + 4\sqrt{2}}{2\sqrt{5} + 4\sqrt{2}}$$

Recall the form $(a + b)(a - b) = a^2 - b^2$

$$\begin{aligned}
&= \frac{((\sqrt{3})^2 - (\sqrt{7})^2)}{(2\sqrt{5})^2 - (4\sqrt{2})^2} \times 2\sqrt{5} + 4\sqrt{2} \\
&= \frac{3 - 7}{20 - 32} \times 2\sqrt{5} + 4\sqrt{2} \\
&= \frac{-4}{-12} \times 2\sqrt{5} + 4\sqrt{2} \\
&= \frac{1}{3} \times (2\sqrt{5} + 4\sqrt{2}) \\
&= \frac{1}{3} \times 2(\sqrt{5} + 2\sqrt{2}) \\
&= \frac{2}{3} \times (\sqrt{5} + 2\sqrt{2})
\end{aligned}$$

- (3). The solution of the equation $2^{x^2-3x-4} = 4^{x+1}$ if p and q are $p > q$, the value of $p - q$ is
- 1
 - 1
 - 5
 - 6
 - 7

Answer:

$$\begin{aligned}
2^{x^2-3x-4} &= (2^2)^{x+1} \\
2^{x^2-3x-4} &= 2^{2x+2} \\
x^2 - 3x - 4 &= 2x + 2 \\
x^2 - 5x - 6 &= 0 \\
(x - 6)(x + 1) &= 0 \\
x - 6 = 0 \cup x + 1 = 0 \\
x = 6 \cup x = -1 \\
\text{Since the value of } p > q, \text{ then } p = 6 \text{ and } q = -1 \\
p - q &= 6 - (-1) \\
&= 7
\end{aligned}$$

- (4). If $\log_8 b = 2$ and $\log_4 d = 1$, the relationship between b and d values is
- $b = \sqrt{d^3}$
 - $b = 3d$
 - $b = \frac{1}{3}d$
 - $b = d^{\frac{1}{3}}$
 - $b = d^3$

Answer:

$$\begin{aligned}
\log_a b = c &\rightarrow a^c = b \\
\log_8 a = 2 &\rightarrow 8^2 = b \\
\log_4 d = 1 &\rightarrow 4^1 = d
\end{aligned}$$



$$\begin{aligned}
 \text{Then, } b &= 8^2 = 64, d = 4^1 = 4 \\
 64 &= 4^3 \\
 64 &= 4 \cdot 4 \cdot 4 \\
 64 &= 64 \\
 b &= d^3
 \end{aligned}$$

- (5). Let p and q be the solution of $\log_2(x+2) - \log_4(3x^2 - x + 6) = 0$. If $p > q$, then the $p - q$ value is
- 3
 - $\frac{5}{2}$
 - $\frac{2}{2}$
 - $\frac{3}{2}$
 - 1

Answer:

$$\begin{aligned}
 \log_2(x+2) - \log_{2^2}(3x^2 - x + 6) &= 0 \\
 \log_2(x+2) - \frac{1}{2} \cdot \log_2(3x^2 - x + 6) &= 0 \\
 \log_2(x+2) - \log_2 \sqrt{3x^2 - x + 6} &= 0 \\
 \log_2 \left(\frac{x+2}{\sqrt{3x^2 - x + 6}} \right) &= 0
 \end{aligned}$$

Recall the definition of logarithm

$$\begin{aligned}
 \log_a b = c &\leftrightarrow a^c = b \\
 \frac{x+2}{\sqrt{3x^2 - x + 6}} &= 1 \\
 (x+2) &= \sqrt{3x^2 - x + 6} \\
 (x+2)^2 &= (\sqrt{3x^2 - x + 6})^2 \\
 x^2 + 4x + 4 &= 3x^2 - x + 6 \\
 2x^2 - 5x + 2 &= 0 \\
 (2x-1)(x-2) &= 0 \\
 x_1 &= \frac{1}{2} \cup x_2 = 2
 \end{aligned}$$

Remember, in the problem $p > q$, then $p = 2$ and $q = \frac{1}{2}$

The value of $p - q$ is $2 - \frac{1}{2} = \frac{3}{2}$

Learning effectiveness is the percentage of achievement of learning objectives which is influenced by many factors, which can be grouped into three domains, namely cognitive, affective, and psychomotor (Abdurahim, 2016). One thing that shows the effectiveness of learning is when students are able to solve HOTS problems, one of which is from several examples of problems above.

4. Conclusion

This paper emphasizes the importance of understanding the concepts of exponents, roots and logarithms in mathematics. This material is an important foundation for various



applications in daily life and in higher mathematics subjects. A strong understanding of these concepts will help learners in solving math problems better.

In addition, this paper also identifies some common errors that often occur when learning exponents, roots and logarithms. These errors include confusion between the concepts and lack of understanding of how to apply them in different contexts. It is important for learners to avoid these mistakes by having a good understanding of these basic concepts.

To assist learners in learning this material, the paper also presents a range of suggested learning media. These include the use of manipulatives, interactive whiteboards, learning videos, and worksheets. These media will help learners to visualize the concepts and apply them in different contexts.

Finally, the paper suggests using UN-type questions (HOTS) to test learners' understanding. These problems are designed to encourage learners to think at a higher level in mathematical problem solving, so that they can better master the concepts of exponents, roots and logarithms.

With a strong understanding of concepts, avoiding common mistakes, using appropriate learning media, and practicing with HOTS questions, learners can achieve a deeper level of understanding and higher-order thinking skills in mathematics. This will help them in facing more complex math challenges in the future.

References

- Abdurahim, A. (2016). Keefektifan model pembelajaran resik ditinjau dari sikap, motivasi, dan kemampuan berpikir kritis matematis siswa SMP. *Jurnal Riset Pendidikan Matematika*, 3(2), 137. <https://doi.org/10.21831/jrpm.v3i2.7994>
- Aksel, A., & Gürman-Kahraman, F. (2014). Video Project Assignments and their Effectiveness on Foreign Language Learning. *Procedia - Social and Behavioral Sciences*, 141, 319–324. <https://doi.org/10.1016/j.sbspro.2014.05.055>
- Amadori, A., Pintore, F., & Sala, M. (2018). On the discrete logarithm problem for prime-field elliptic curves. *Finite Fields and Their Applications*, 51, 168–182. <https://doi.org/10.1016/j.ffa.2018.01.009>
- Bui, P., Pongsakdi, N., McMullen, J., Lehtinen, E., & Hannula-Sormunen, M. M. (2023). A systematic review of mindset interventions in mathematics classrooms: What works and what does not? *Educational Research Review*, 40(July), 100554. <https://doi.org/10.1016/j.edurev.2023.100554>
- Chung, K. (2023). Factorization of coefficients for exponential and logarithm in function fields. *Journal of Number Theory*, 245, 263–291. <https://doi.org/10.1016/j.jnt.2022.11.001>
- Hong, J., & Lee, H. (2019). Solving discrete logarithm problems faster with the aid of pre-computation. *Discrete Applied Mathematics*, 267, 93–119. <https://doi.org/10.1016/j.dam.2019.03.023>
- Larsson, M., & Ruf, J. (2019). Stochastic exponentials and logarithms on stochastic intervals — A survey. *Journal of Mathematical Analysis and Applications*, 476(1), 2–12. <https://doi.org/10.1016/j.jmaa.2018.11.040>
- Renata, B., & Jana, M. (2012). Learning and Teaching with Technology E-learning as a Motivation in Teaching Physics. *Procedia - Social and Behavioral Sciences*, 64, 328–331. <https://doi.org/10.1016/j.sbspro.2012.11.039>
- Su, J., & Yang, W. (2023). A systematic review of integrating computational thinking in early childhood education. *Computers and Education Open*, 4(December 2022), 100122. <https://doi.org/10.1016/j.cao.2023.100122>
- Susanto, D., 2021. *Buku Panduan Guru Matematika*. 1 ed. Jakarta Pusat: Pusat Kurikulum dan Perbukuan.
- Susanto, D., 2021. *Matematika*. 1 ed. Jakarta Pusat: Pusat Kurikulum dan Perbukuan.